F=MA Competition Equation Sheet: Version 1.3

1 Kinematics

$$x = x_0 + vt$$

$$v_{avg} = \frac{\Delta x}{t}$$

$$a_{avg} = \frac{\Delta v}{t}$$

Constant acceleration:

$$v = v_0 + at$$

$$x = \frac{1}{2}(v_0 + v_f)t$$

$$x = v_0 t + \frac{1}{2}at^2$$

$$x = v_f t - \frac{1}{2}at^2$$

$$v_f^2 = v_0^2 + 2ax$$

Changing acceleration:

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Projectile motion (only use the first three if $\Delta y = 0$) :

$$T_{flight} = \frac{2v_0 sin\theta}{g}$$

$$Height_{max} = \frac{v_0^2 sin^2\theta}{2g}$$

$$Range = \frac{v_0^2 sin2\theta}{g}$$

$$a_x = 0$$

$$a_y = g = -10\ \frac{m}{s^2}$$

$$v_x = v_0 cos\theta$$

$$v_y = v_0 sin\theta$$

$$x = v_x t$$

$$y = xtan\theta - \frac{gx^2}{2v_0^2 cos^2\theta}$$

$$Time\ to\ drop\ object : \sqrt{\frac{2h}{g}}$$

$$Addition\ of\ relative\ velocity : v_{ac} = v_{ab} + v_{bc}$$

$$a_{ac} = a_{ab} + a_{bc}$$

2 Dynamics

$$if\ \Sigma F = 0,\ a = 0$$

$$\Sigma \overrightarrow{F} = m\overrightarrow{a}$$

$$\overrightarrow{F_{ab}} = \overrightarrow{-F_{ba}}$$

$$F_c = ma_c = \frac{mv^2}{r} = m\omega^2 r = m\frac{4\pi r^2}{T^2}$$

$$F_g = mg$$

$$f_s \leq \mu_s F_N$$

$$f_k = \mu_k F_N$$

$$F_{hookian\ spring} = -k\Delta x$$

$$k_p = \Sigma_i k_i$$

$$k_s = (\Sigma_i \frac{1}{k_i})^{-1}$$

$$F_N = mgcos\theta$$

$$tan\theta_{slip} = \mu$$

$$F_{coriolis} = -2m\omega \times v_{object}$$

Massless pulleys:

$$a = \frac{m_2 - m_1}{m_2 + m_1} g$$

$$F_T = \frac{2m_1 m_2}{m_1 + m_2} g$$

Drag:

$$Linear\ drag : v_{terminal} = \frac{mg}{b}$$

$$Quadratic\ drag : v_{terminal} = \sqrt{\frac{mg}{b}}$$

The Galilean Transformations

Consider two reference frames S and S′. The coordinate axes in S are *x*, *y*, *z* and those in S′ are *x*′, *y*′, *z*′. Reference frame S′ moves with velocity *v* relative to S along the *x*-axis. Equivalently, S moves with velocity −v relative to S′.

The *Galilean transformations of position* are:

$$x = x' + vt \qquad x' = x - vt$$
$$y = y' \qquad \text{or} \qquad y' = y$$
$$z = z' \qquad z' = z$$

The *Galilean transformations of velocity* are:

$$u_x = u'_x + v \qquad u'_x = u_x - v$$
$$u_y = u'_y \qquad \text{or} \qquad u'_y = u_y$$
$$u_z = u'_z \qquad u'_z = u_z$$

$$a'_x = a_x \quad (\because \text{velocity v is constant})$$
$$a'_y = a_y, \quad a'_z = a_z$$
$$\therefore a' = a$$

$$\mathbf{F}' = \mathbf{F}$$

3 Energy

$$W = F \cdot d = \int F(x)dx$$

$$K = \frac{1}{2}mv^2$$

$$W = \Delta K = \frac{1}{2}m(v_f^2 - v_0^2)$$

$$U_g = mgh$$

$$U_{hookian\ spring} = \frac{1}{2}kx^2$$

$$U = -\int F(x)dx$$

$$W_{field} = -\Delta U$$

$$E_{dis} = fd$$

$$E_{mech} = K + U$$

$$E_i - E_{dis} = E_f$$

$$P = F \cdot v$$

$$P = \frac{dE}{dt}$$

4 System of masses and collisions

$$p = mv$$

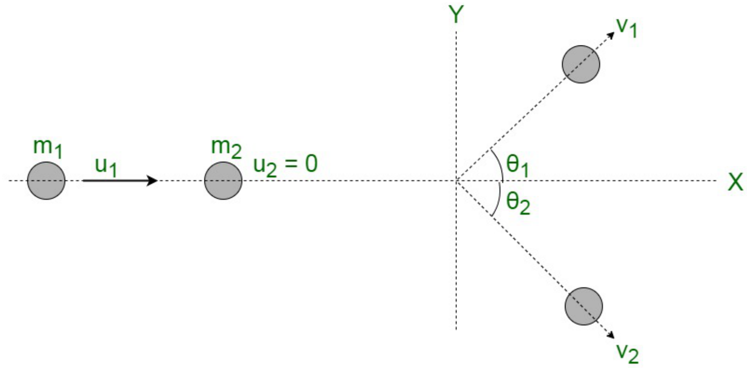
$$j = Ft = m\Delta v = \Delta p$$

$$F = \frac{dp}{dt}$$

$$\overrightarrow{p_{1,i}} + \overrightarrow{p_{2,i}} = \overrightarrow{p_{1,f}} + \overrightarrow{p_{2,f}}$$

$$inelastic\ collision: v_f = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

$$elastic\ collision: v_1 = \frac{(m_1 - m_2)v_{1,i} + 2m_2v_{2,i}}{m_1 + m_2},\ v_2 = \frac{-(m_1 - m_2)v_{2,i} + 2m_2v_{1,i}}{m_1 + m_2}$$



If m1=m2, and the collision is elastic, then:

$$\theta_1 + \theta_2 = 90\ degrees$$

$$x_{com} = \frac{\Sigma mr}{M}\ or\ \frac{1}{M}\int rdm$$

$$v_{com} = \frac{\Sigma mv}{M} or\ \frac{1}{M}\int vdm$$

$$Densities : \lambda = \frac{M}{L}, \sigma = \frac{M}{L^2}, \rho = \frac{M}{L^3}$$

5 Rotational motion, Rigid Bodies

$$if\ \Sigma\tau = 0,\ \alpha = 0$$

$$x = r\theta$$

$$v = r\omega$$

$$a = r\alpha$$

$$\theta = \theta_0 + \omega t$$

$$\omega_{avg} = \frac{\Delta\theta}{t},\ \alpha_{avg} = \frac{\Delta\omega}{t}$$

$$\omega = \frac{d\theta}{dt}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

$$\omega = \omega_0 + \alpha t,\ x = \frac{1}{2}(\omega_0 + \omega_f)t$$

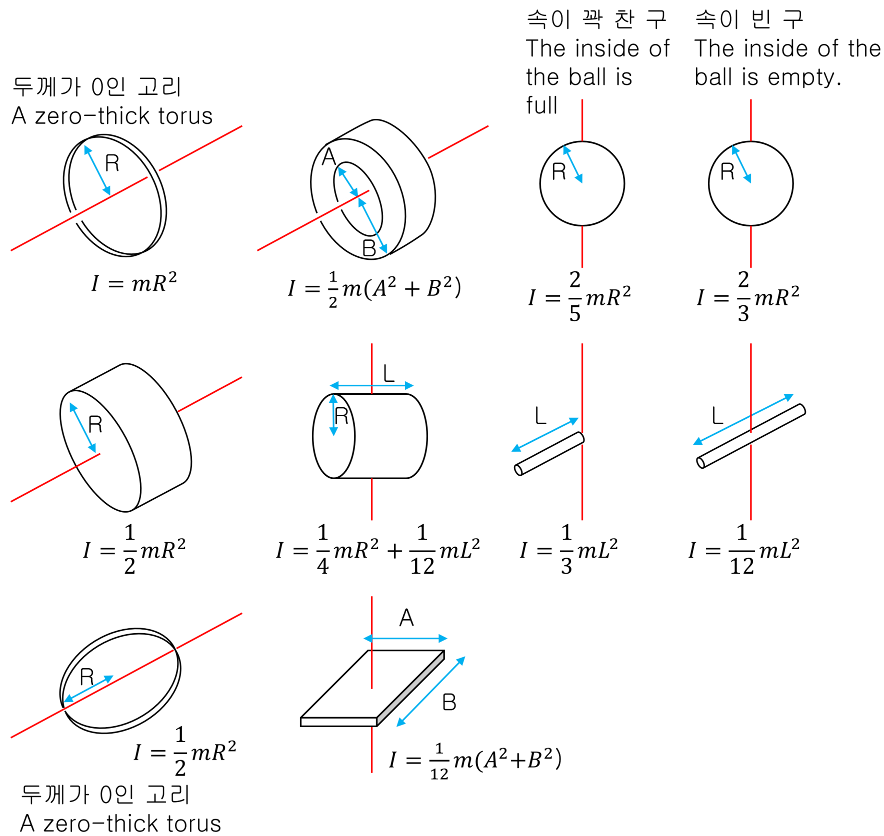
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2,\ \theta = \omega_f t - \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\theta$$

$$\tau = r \times F,\ \tau = I\alpha$$

β is the distribution of mass:

$$I = \int r^2 dm,\ I_{discrete} = \Sigma_{i=1}^{N} m_i r_i^2,\ I = \beta m r^2$$



$$I = I_{cm} + Md^2$$

$$I_z = I_x + I_y$$

$$I \propto L^{d+2}$$

$$K_{rot} = \frac{1}{2}I\omega^2,\ K_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$W = \tau\theta,\ P = \tau\omega$$

$$\tau = \frac{dL}{dt},\ \Delta L = \tau t$$

$$L_{point} = mvr,\ L_{fixed-pivot} = I\omega$$

$$L = r \times p,\ \Delta L = r\Delta p$$

$$a_{rolling\ down\ incline} = \frac{g sin\theta}{1+\beta}$$

$$tan\theta_{slip} = \mu(1+\frac{1}{\beta})$$

$$a_{roll\ and\ slip} = g(sin\theta - \mu cos\theta)$$

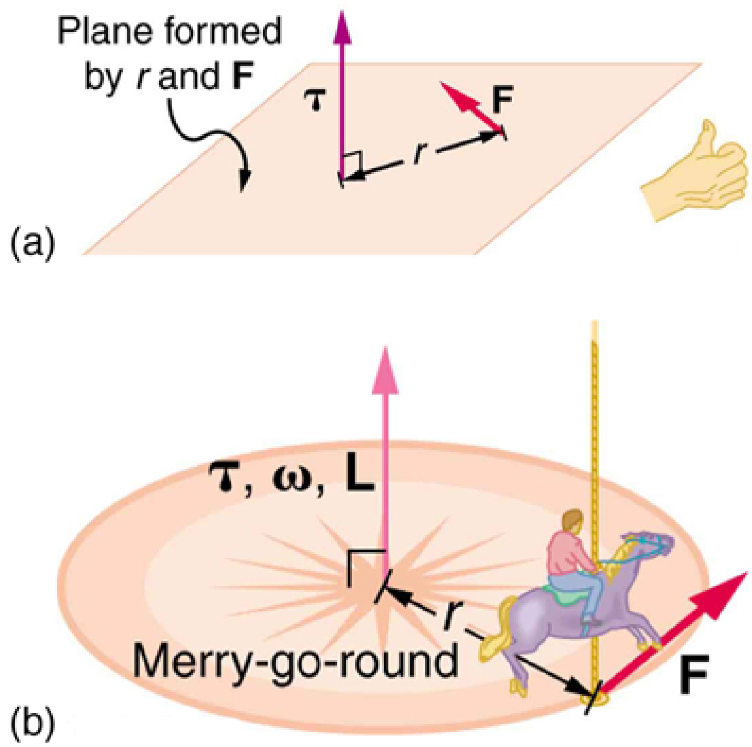
$$\alpha_{roll\ and\ slip} = \frac{\mu g cos\theta}{\beta r}$$

$$pulley\ mass\ not\ negligible : a = \frac{(M-m)g}{M+m+\beta M_p}$$

Final velocity for rolling on friction surface:

$$v_f = r\omega_f = \frac{v_i}{1+\beta}$$

Right Hand Rule for angular motion:



6 Oscillatory Motion

$$f = \frac{1}{T}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$E_{total} = K + U = \frac{1}{2}kA^2$$

$$T_{pendulum} = 2\pi\sqrt{\frac{L}{g}},\ \omega_{pendulum} = \sqrt{\frac{g}{L}}$$

$$T_{spring} = 2\pi\sqrt{\frac{m}{k}},\ \omega_{spring} = \sqrt{\frac{k}{m}}$$

$$T_{physical} = 2\pi\sqrt{\frac{I_{support}}{mgL_{cm}}},\ \omega_{physical} = \sqrt{\frac{mgL_{cm}}{I_{support}}}$$

$$T_{torsion} = 2\pi\sqrt{\frac{I}{\kappa}},\ \omega_{torsion} = \sqrt{\frac{\kappa}{I}}$$

$$\omega_{damping} = \sqrt{\frac{k}{m} - (\frac{b}{2m})^2},\ ma = -kx - bv$$

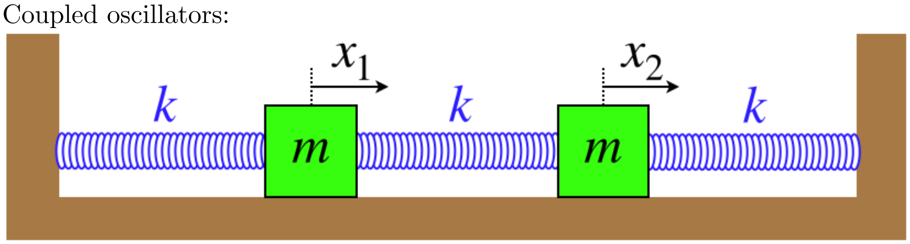
$$\ddot{x} = -\omega^2 x$$

$$x = Acos(\omega t + \phi)$$

$$v = -A\omega sin\omega t, v_{max} = A\omega$$

$$a = -A\omega^2 cos\omega t,\ a_{max} = A\omega^2$$

Coupled oscillators:



If in phase:

$$\omega = \sqrt{\frac{m}{k}}$$

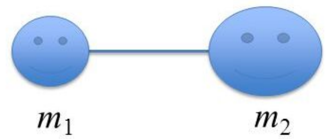
if not in phase:

$$\omega = \sqrt{\frac{m}{3k}}$$

For reduced mass oscillation:

- μ is the "reduced mass":

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$



- k is the "spring constant"
  - Measures "stiffness" of the bond

$$\omega = \sqrt{\frac{k}{\mu}}$$

With the spring constant and reduced mass we can obtain fundamental vibrational frequencies

7 Gravity

$$\Phi_g = -4\pi G M_{enc}$$

$$F_{g,\ sphere} = \frac{GMm}{r^2},$$

$$g_{sphere} = \frac{GMr}{R^3},\ g_{shell} = 0,\ r < R$$

$$g_{sphere} = g_{shell} = \frac{GM}{r^2},\ r > R$$

$$For\ a\ infinite\ line:\ g = \frac{2G\lambda}{d}$$

$$For\ a\ infinite\ plane:\ g = 2\pi G\sigma$$

For a infinite cylinder with radius a:

$$g_{cylinder} = \frac{2G\lambda}{r},\ r > a$$

$$g_{cylinder} = \frac{2G\lambda r}{a^2},\ r < a$$

For a infinite plane with thickness a:

$$g = 2\pi G\sigma,\ d > a$$

$$g = \frac{2\pi G\sigma d}{a},\ d < a$$

$$U_g = -\frac{GMm}{r}$$

$$\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$$

$$T_{orbit} = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = 2\pi R\sqrt{\frac{R}{GM}}$$

$$v_a r_a = v_p r_p$$

$$\frac{dA}{dt} = \frac{1}{2}rv = \frac{L}{2m} = constant$$

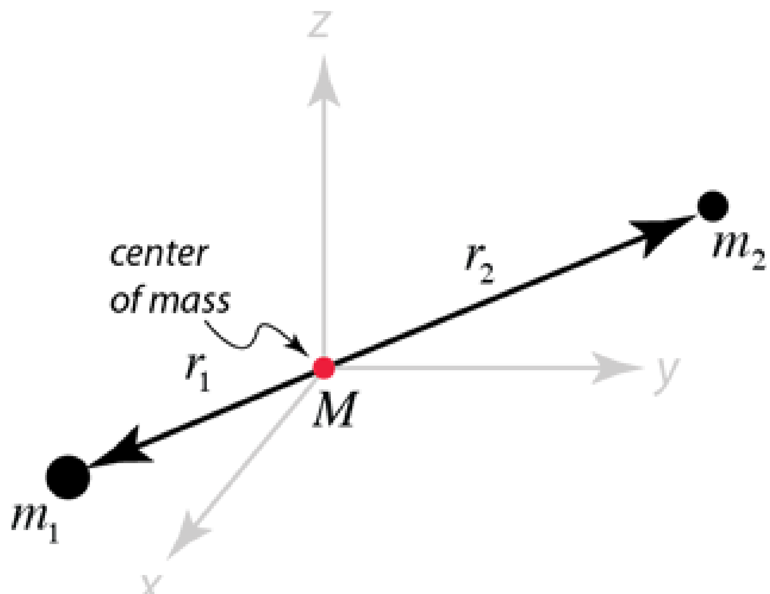
$$v_{escape} = \sqrt{\frac{2GM}{r}}$$

$$v_{circ} = \sqrt{\frac{GM}{r}},\ E_{circ} = \frac{-GMm}{2r}$$

$$Gravitational\ binding\ energy: U = \frac{-3GM^2}{5R}$$

$$R_{Black\ Hole} = \frac{2GM}{c^2}$$

Reduced Mass(convert a two body problem to a one body problem):



$$Center\ mass = m_1 + m_2$$

$$orbiting\ mass = \frac{m_1 m_2}{m_1 + m_2}$$

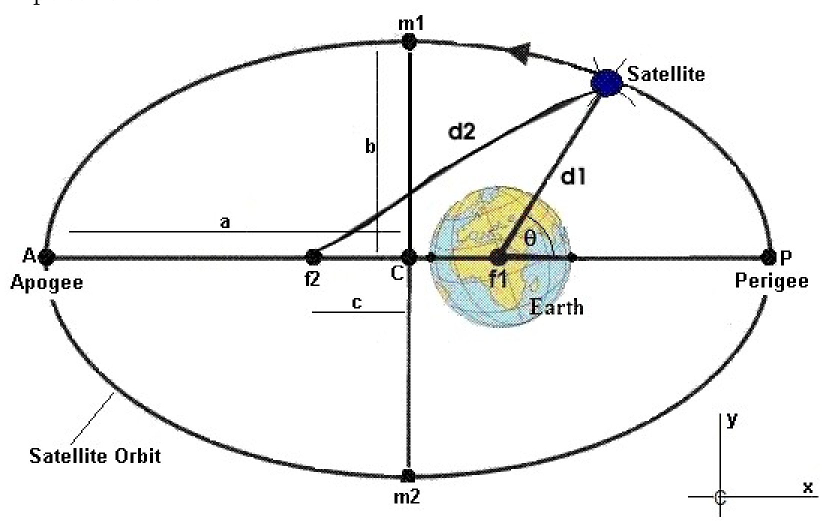
$$r = r_1 + r_2$$

$$\frac{T^2}{(r_1 + r_2)^3} = \frac{4\pi^2}{G(m_1 + m_2)}$$

$$T = T_1 = T_2 = 2\pi(r_1 + r_2)\sqrt{\frac{r_1 + r_2}{G(m_1 + m_2)}}$$

$$\frac{Gm_1m_2}{(r_1 + r_2)^2} = \frac{m_1v_1^2}{r_1} = \frac{m_1v_1^2}{r_2}$$

Elliptical orbit:



$$d_1 + d_2 = 2a, \; A = \pi ab$$

$$a^2 = b^2 + c^2$$

$$v_{ellip} = \sqrt{GM(\frac{2}{r} - \frac{1}{a})}, \; E_{ellip} = \frac{-GMm}{2a}$$

$$v_{apogee} = \frac{a - c}{b}\sqrt{\frac{GM}{a}} = \sqrt{\frac{a - c}{a + c}}\sqrt{\frac{GM}{a}}$$

$$v_{perigee} = \frac{a + c}{b}\sqrt{\frac{GM}{a}} = \sqrt{\frac{a + c}{a - c}}\sqrt{\frac{GM}{a}}$$

$$v_{co-vertex} = \sqrt{\frac{GM}{a}}$$

$$L_{ellip} = mb\sqrt{\frac{GM}{a}}$$

$$T_{ellip} = 2\pi\sqrt{\frac{a^3}{GM}}$$

8 Fluid mechanics and Other Topics

$$\rho = \frac{m}{V}$$

$$F = PA$$

$$F_1 A_1 = F_2 A_2$$

$$P = P_0 + \rho g h,\ P_{gauge} = \rho g h$$

$$F_b = \rho_{fluid} V_{submerged} g$$

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$v = \sqrt{2gh}$$

$$t_{drain\ tank} = \frac{A_{tank}}{A_{hole}}\sqrt{\frac{2h}{g}}$$

$$Y = \frac{tensile\ stress}{tensile\ strain} = \frac{F}{A}\frac{L}{\Delta L}$$

$$S = \frac{shear\ stress}{shear\ strain} = \frac{F}{A}\frac{h}{\Delta x}$$

$$B = \frac{volume\ stress}{volume\ strain} = -\frac{\Delta F}{A}\frac{V}{\Delta V} = -\Delta P \frac{V}{\Delta V}$$

$$spring\ constant\ k = \frac{AY}{L_0}$$

Waves:

$$y = Asin(kx - \omega t + \phi)$$

$$v = \lambda f = \frac{\omega}{k}$$

wave number: $k = \frac{2\pi}{\lambda}$, angular frequency: $\omega = 2\pi f = \frac{2\pi}{T}$

$$\lambda_n = \frac{2L}{n},\ f_n = \frac{nv}{2L}$$

$$v_{string} = \sqrt{\frac{TL}{M}},\ v_{sound} = \sqrt{\frac{B}{\rho}}$$

$$x_{node} = \frac{n\lambda}{2},\ x_{antinode} = \frac{(2n-1)\lambda}{4}$$

$$f' = f(\frac{v\ \pm v_o}{v\ \mp v_s})$$

Error propagation:

Process	**Value**	**Uncertainty**
Average	$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_N}{N}$	$\sigma_x = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_N - \bar{x})^2}{N-1}}$
Addition	$\bar{z} = \bar{x} + \bar{y}$	$\sigma_z = \sqrt{(\sigma_x)^2 + (\sigma_y)^2}$
Subtraction	$\bar{z} = \bar{x} - \bar{y}$	$\sigma_z = \sqrt{(\sigma_x)^2 + (\sigma_y)^2}$
Multiplication	$\bar{z} = \bar{x} * \bar{y}$	$\sigma_z = \bar{z} * \sqrt{\left(\frac{\sigma_x}{\bar{x}}\right)^2 + \left(\frac{\sigma_y}{\bar{y}}\right)^2}$
Division	$\bar{z} = \bar{x} \div \bar{y}$	$\sigma_z = \bar{z} * \sqrt{\left(\frac{\sigma_x}{\bar{x}}\right)^2 + \left(\frac{\sigma_y}{\bar{y}}\right)^2}$

Exponential:

$$\mathbf{Q = a^n \rightarrow \frac{\Delta Q}{Q} = \left| n \frac{\Delta a}{a} \right|}$$

$$\Delta f = Uncertainty\ of\ measurement : f(x,y) = cx^n y^m$$

$$\frac{\Delta f}{f} = \sqrt{(\frac{n\Delta x}{x})^2 + (\frac{m\Delta y}{y})^2}$$

Ways to eliminate answer choices:

Dimensional analysis: find the units of each answer choice and match it with what the question asks you to find.

Limiting cases: imagine a parameter is either extremely big, small, or is a special number that you would intuitively try.

9 Further notes

- A dot product $\cdot$ is a way to multiply two vectors, where $X \cdot Y = XY cos\theta$
- A cross product $\times$ is another way to multiply two vectors, where $X \times Y = XY sin\theta$
- A derivative is a rate of change, which is the slope of a graph in geometric terms. For example:

$$v = \frac{dx}{dt}$$

  You can think of velocity as the slope of the displacement vs. time graph.
- An integral is the space under a graph of an equation, the opposite of a derivative. For example:

$$W = \int F(x)dx$$

  You can think of work as the area under the curve of the force vs. distance graph.
- The notation $\ddot{x}$ represent $\frac{d^2x}{dt^2}$.
- Always draw a free body diagram when doing dynamics problems!