Notation

O.P.: Orbital Pole (Ecliptic)
N.C.P.: Northern Celestial Pole (Celestial)
Z: Zenith (Local, alt-azimuth)
$\odot$: the Sun, Star
$\oplus$: the Earth, Planet
ϵ: obliquity, axial tilt, 23.45^o
λ: longitude
ϕ: latitude
α: RA, Right Ascension
h: Hour Angle
δ: declination
A: Azimuth
a: altitude
Θ: Sidereal time (hour angle of vernal equinox)
S: Synodic time
$T_\odot$: solar or synodic day (361^o)
$T_\oplus$: sidereal day (360^o)
$P_\oplus$: Earth orbital period (1 year)
P: planet orbital period
m: apparent magnitude
M: absolute magnitude

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v_∞: hyperbolic excess velocity
D_C: comoving distance
D_A: Angular diameter distance
D_L: Luminosity distance
D_P: Proper distance
T_{MS}: Mean solar time
T_S: Solar time
GST: Greenwich (mean) sidereal time
UT: Universal Time (UT1)
CT: Civil Time

Astronomy

$$360^o = 24hours, 15^o = 1hour,$$
$$1^o = 4minutes, 1^o = 60' = 3600"$$

$$O.P. - N.C.P. = 23.5^o$$

$$Zenith - N.C.P. = 90^o - \phi$$

$$s = R\theta$$

$$1parsec = \frac{1AU}{1"}$$

Zodiac: Aries, Taurus, Gemini (July), Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius (January), Capricorn, Aquarius and Pisces (VE). 2h every month. CCW: increasing RA.

Moon Phases: new Moon, waxing crescent, first quarter, waxing gibbous, full Moon, waning gibbous, third quarter and waning crescent

$$\omega_1 - \omega_2 = \Delta\omega$$

$$\frac{1}{T_{sidereal}} - \frac{1}{T_{synodic}} = \frac{1}{T_{orbital}}$$

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{P_\oplus} \text{ [inferior]}$$

$$\frac{1}{S} = \frac{1}{P_\oplus} - \frac{1}{P} \text{ [superior]}$$

Spherical Geometry

$$E = A + B + C - \pi$$
$$Area = ER^2$$
$$distance = \arccos(\sin(\phi_1)\sin(\phi_2) + \cos(\phi_1)\cos(\phi_2)\cos(\Delta lon))$$
$$*[1]\ \cos c = \cos a \cos b + \sin a \sin b \cos C$$
$$*\cos C = -\cos A \cos B + \sin A \sin B \cos c$$
$$*\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$
$$*\cos B \sin a = -\cos A \sin b \cos c + \cos b \sin c$$

Positional Astronomy

$$\cos h = -\tan\delta \tan\phi \text{ [sunrise, sunset]}$$
$$\Theta = h + \alpha$$
$$*A_{\text{shadow}} = A_{\text{gnomon}} \cot a_\odot \sin A_\odot$$
$$*\sin h \cos\delta = \sin A \cos a$$
$$*\cos h \cos\delta = \cos A \cos a \sin\phi + \sin a \cos\phi$$
$$*\sin\delta = \sin\phi \sin a - \cos\phi \cos a \cos A$$
$$*\sin A \cos a = \sin h \cos\delta$$
$$*\cos A \cos a = \cos h \cos\delta \sin\phi - \sin\delta \cos\phi$$
$$*\sin a = \sin\phi \sin\delta + \cos\phi \cos\delta \cos h$$
$$*\Theta \approx T_{MS} + 12h + n_{days} * 4min = \alpha + h$$
$$*T_S = T_{MS} + E.T. \approx T_{MS} \pm 15\ min$$
$$*GST = GST_0 + \frac{n_{days}}{365.2422\ \ days} \cdot 24\ \ hours + \frac{24}{23.9344} \cdot \Delta UT$$

Stars

$$L = P = \sigma T^4 A = \sigma T^4 4\pi R^2$$
$$I = R = F = B = \frac{L}{4\pi d^2} \propto \frac{1}{d^2}$$
$$M_A - M_B = -2.5\log_{10}(\frac{L_A}{L_B})$$
$$m_A - m_B = -2.5\log_{10}(\frac{I_A}{I_B})$$
$$m - M = 5\log_{10}(d) - 5$$
$$P_G(r=0) = \frac{2}{3}\pi G\rho^2 R^2$$
$$\text{Transit Depth} = \frac{R_p^2}{R_s^2}$$
OBAFG(Sun)KMTN
$$*M = -2.78[\log(T) - 1] - 4.00$$
$$*L \propto \text{Mass}^{3.5}$$
$$*E = \text{Mass} \cdot c^2$$
$$*\text{longivity} \propto \text{Mass}^{-2.5}$$

Cosmology

$$v_r = H_0 d$$

$$\lambda = \lambda_0(1 + \frac{v_r}{c})$$

$$z = \frac{\Delta\lambda}{\lambda} = \frac{f_{emission}}{f_{obs}} - 1 = \frac{v_r}{c}$$

$$age = \frac{1}{H_0}$$

$$R_{\text{Schwarzschild}} = \frac{2GM}{c^2}$$

$$\rho_0 = \frac{3H_0^3}{8\pi G}$$

$$*D_L = (1+z)D_C = (1+z)^2 D_A = (1+z)^2 D_P$$

$$*G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$*\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \frac{3p}{c^2}) + \frac{\Lambda c^2}{3}$$

$$*H_0^2 = (\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$*f_{grav} = 2f_{orbit}$$

$$*P_{quantum} = \frac{\pi^3\hbar^2}{15m}(\frac{3n}{\pi})^{\frac{5}{3}}$$

Ellipses

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$c^2 = a^2 - b^2, e = \frac{c}{a}, A = \pi ab$$

$$\text{apogee} = a + c, \text{perigee} = a - c$$

$$\frac{dy}{dx} = -\frac{b^2}{a^2}\frac{x}{y}, -\frac{dx}{dy} = \frac{a^2}{b^2}\frac{y}{x}$$

$$x = a\cos\theta, y = b\sin\theta$$

$$r(\theta) = \frac{a(1-e^2)}{1 \pm e\cos\theta}$$

[1] Equations marked with (*) are only required for second round of Astro Olympiad.

Gravity

$$F_g = G\frac{Mm}{r^2}$$

$$g = \frac{GM}{r^2}$$

$$U = -\frac{GMm}{r}$$

$$E_{total} = -\frac{GMm}{2a}$$

$$v_{esc} = v_{parabola} = \sqrt{\frac{2GM}{r}}$$

$$\frac{T^2}{a^3} = \frac{4\pi^2}{GM}$$

$$v(r) = \sqrt{GM(\frac{2}{r} - \frac{1}{a})}$$

$$-2 < KE >=< PE >$$

$$*F_{tidal} = \frac{2GMm}{R^3}\Delta R$$

$$*U_{self,sphere} = -\frac{3}{5}\frac{GM^2_{sphere}}{r}$$

$$*v^2_{perigee} = v^2_{esc} + v^2_{\infty}$$

Thermaldynamics

$$pV = nRT = Nk_BT$$

$$\overline{KE_t} = 3/2\ k_B\ T$$

$$\lambda_{max}T = b$$

$$T_{planet,eq} = \sqrt[4]{\frac{(1-a)R^2_{\odot}}{4d^2}}T_{\odot}$$

$$*I(f,T) = \frac{2hf^3}{c^3}\frac{1}{e^{\frac{hf}{k_BT}} - 1}$$

$$*I(\lambda,T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_BT}} - 1}$$

Telescope

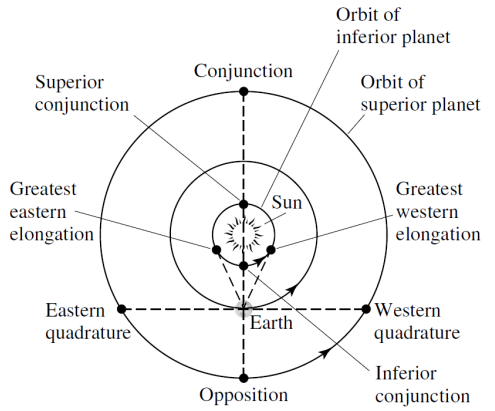
$$v = f\lambda$$

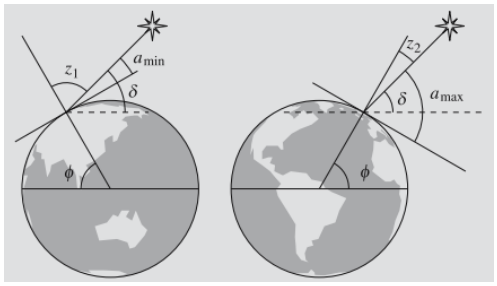
$$\theta = 1.22\frac{\lambda}{D} \approx \frac{\lambda}{D}$$

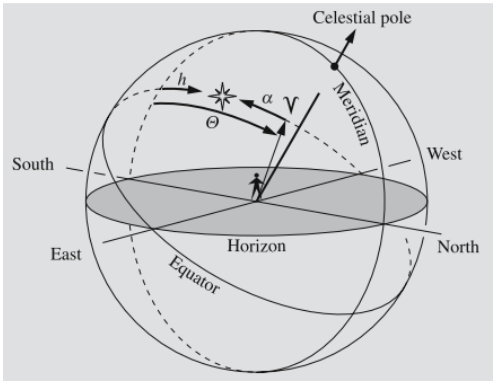
$$f/n: \quad f = nD$$

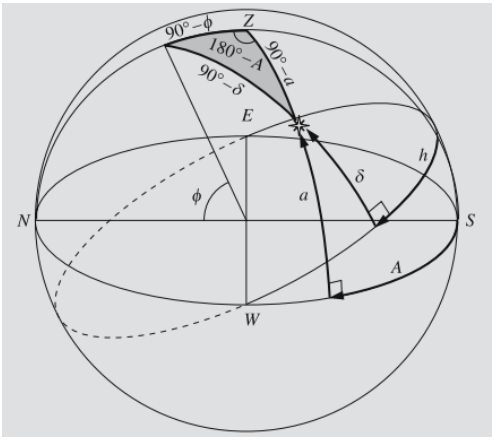
$$M = -\frac{f_{obj}}{f_{eye}} \approx \frac{D_{obj}}{D_{eye}}$$

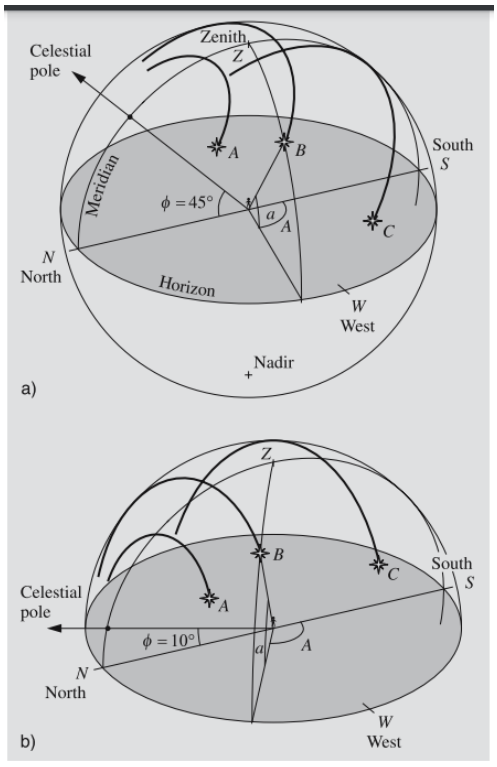
$$*TFOV = \frac{AFOV}{\text{magnification}}$$

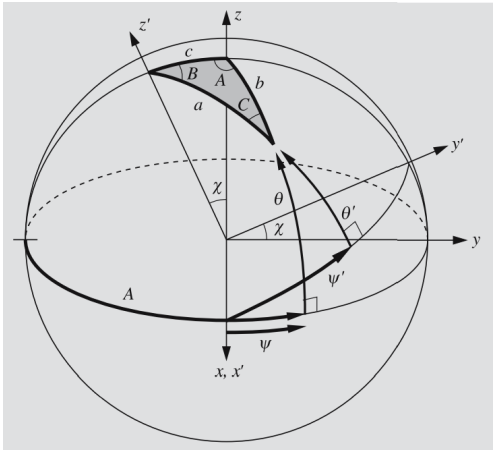












USAAAO 2021 - First Round

January 30^{th}, 2021

PHYSICAL AND ASTRONOMICAL CONSTANTS

c	Speed of light in vacuum	2.998×10^8m s^{-1}
e	Elementary charge	1.602×10^{-19} C
m_n	Neutron rest mass	1.675×10^{-27} kg
m_p	Proton rest mass	1.6725×10^{-27} kg
m_e	Electron rest mass	9.110×10^{-31} kg
m_{He}	Helium-4 rest mass	6.644×10^{-27} kg
h	Planck's constant	6.626×10^{-34} J s
H_0	Hubble's constant	70 (km/s)/Mpc
k_B	Boltzmann's constant	1.381×10^{-23} J K^{-1}
b	Wien's constant	2.898×10^{-3} m K
G	Gravitational constant	6.673×10^{-11} N m^2 kg^{-2}
σ	Stefan-Boltzmann constant	5.670×10^{-8} J m^{-2} K^{-4} s^{-1}
c_1	First Radiation Constant ($= 2\pi hc^2$)	3.742×10^{-16} J m^2 s^{-1}
c_2	Second Radiation Constant ($= hc/k$)	1.439×10^{-2} m K
N_A	Avogadro constant	6.022×10^{23} mol^{-1}
R	Gas constant	8.314 J K^{-1} mol^{-1}
a_0	Bohr radius	5.292×10^{-11} m
μ_B	Bohr magneton	9.274×10^{-24} J T^{-1}
$M_\odot$	Solar mass	1.989×10^{30} kg
$R_\odot$	Solar radius	6.96×10^8 m
$L_\odot$	Solar luminosity	3.827×10^{26} J s^{-1}
$T_\odot$	Solar temperature	5770 K
$M_\oplus$	Earth mass	5.976×10^{24} kg
$R_\oplus$	Mean Earth radius	6.371×10^6 m
$I_\oplus$	Earth moment of Inertia	8.04×10^{37} kg m^2
$R_☾$	Mean Moon radius	1.737×10^6 m
$M_♃$	Mean Jupiter mass	1.9×10^{27} kg
$R_♃$	Mean Jupiter radius	7.1492×10^7 m
$a_♃$	Mean orbital radius of Jupiter	5.2 AU
$a_☾$	Mean semimajor axis Moon orbit	3.84399×10^8 m
1 light year		9.461×10^{15} m
1 AU	Astronomical Unit	1.496×10^{11} m
1 pc	Parsec	3.086×10^{16} m
1 year		3.156×10^7 s
1 sidereal day		86164 s
1 erg		1×10^{-7} J
1 bar		10^5 N m^{-2}

ENERGY CONVERSION : 1 joule (J) = 6.2415×10^{18} electronvolts (eV)